Fast Calculation of the Core Flux for a Transformer with Increased Leakage Inductance

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The particular class of two-winding transformers is considered, for which the knowledge of the core flux density under load condition plays the key role. The standard field-circuit model of a transformer does not give reliable answer due to computational difficulties and physical uncertainty of leakage inductance of two coils having a strong magnetic coupling.

The reformulation of well-known equivalent circuit is proposed, in such a way that all the inductances in it have the clear physical sense in order to be directly measured or modelled by FEA. The results of 2D and 3D FEA simulation compared to each other and with the measured data. It is shown that for transformer design purpose the 2D simulation gives sufficient accuracy. The proposed model is also applicable for a multi-winding transformer used for energy efficient arc welding invertors.

Index Terms— **transformer cores, magnetic flux leakage, equivalent circuits, finite element analysis.**

I. Introduction

Sizing of the magnetic core for power transformers is usually based on no-load conditions. It is quite reasonable for the large majority of power transformers due to low leakage inductance, resulting in a magnetic flux density under load condition is not much different from those under idle run.

Nevertheless there is a class of transformers for which the above assumption is not true. For example, the transformer supplying a resistance-welding machine will never be energized at idle. The working condition of such transformer is similar to the short circuit with small RL load. In order to size the magnetic core properly, one has to know the flux density in the core under load condition.

The new method for calculating the magnetic core flux at load condition is proposed. It is based on classical transformer equations [1], and relies of the separate calculation of two magnetic fields: the main field mainly concentrated in the core, and the leakage field, located mostly outside of the core. The non-linearity is taken into account by the calculation the main field, while the leakage field appears to be almost linear. It is uneasy to use classical transformer equation directly, because it operates with leakage inductance of each winding, which is difficult for measuring and simulating. On the other hand, the short-circuit inductance of any pair of windings can be conveniently measured and modelled by FEA. We rewrite the transformer equations in such a way that the core flux is expressed in terms of two pairs of short-circuit inductances of a three-winding transformer. These equations correspond to the equivalent circuits introduced in **Error! Reference source not found.** and **Error! Reference source not found.**, which were designed primary for transient analysis in a wide frequency range.

The advantage of the transformer equations and the equivalent circuit proposed in this paper is its ability to predict the flux density in given cross-section of the magnetic core under arbitrary load condition. Such knowledge is needed for properly sizing of a power supply for contact welding machines.

II. Core Flux Equation

Our goal is finding the flux density in the given cross-section of the transformer core by known load impedance. Let's label the primary winding as 1, the secondary winding as 2, and mentally place the small imaginary measuring coil 3 just around the place of interest of the magnetic core. Now we can consider a three-winding transformer 1-2-3. The flux linked with a measuring coil 3 expresses in terms of the applied voltage U_1 and the load current I_2:

$$\Phi_3 = -\frac{\dot{E}_{3idle}}{j\omega w_3}\left[1-\frac{w_3}{w_2}\frac{1}{z_{sc}+z_{load}}\left(j\omega M_{\sigma 123}+\frac{r_1}{k_{12}k_{13}}\right)\right], \quad (1)$$

where $\omega=2\pi f$ is the cyclic frequency, j is imaginary unit, E_{3idle} is the no-load voltage of the winding 2, $M_{\sigma123}$ is the equivalent mutual inductance on leakage fluxes, which depends on leakage inductances of transformers 1-2; 1-3 and 2-3; r_1 is the resistance of primary winding, k_{12} and k_{23} are winding ratios, z_{load} is the load impedance, z_{sc} is the short-circuit impedance of the transformer 1-2, related to the winding 2.

Only the E_{3idle} in (1) depends on the core saturation. It can be easily identified by static nonlinear FEA model.

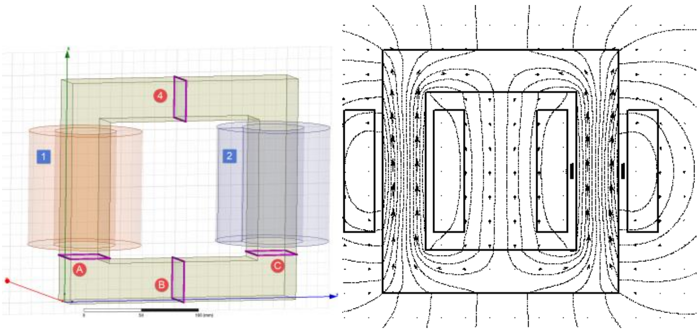
III. Evaluation of the Equivalent Mutual Inductance

The equation (1) introduces the equivalent mutual leakage inductance $M_{\sigma123}$ that is a linear combination of standard leakage inductances of two-winding transformers 1-2, 2-3 and 1-3, related to the primary winding:

$$M_{\sigma123} = \frac{L_{12}+L_{13}-L_{23}}{2k_{12}k_{13}}, \quad (2)$$

Each of three leakage inductances L_{12}, L_{13}, and L_{23} in (3) is easily obtainable by 3D or even 2D FEA simulation. We never consider the leakage inductance of a single winding due to its physical and computational uncertainty. Instead, each term in (2) is the leakage inductance of a pair of windings, which is energized oppositely, i.e. by currents with zero sum. The magnetostatic FEA formulation can used when the skin effect is negligible, or the AC magnetics formulation otherwise. We have compared the 3D and 2D model, both of magnetostatic formulation. For the later model, we choose the average length of coil turns as a model depth.

The field patterns of the primary and secondary windings energized in opposite are shown on fig. 1b and 2.



a) b)

Fig 1. Model Transformer with 3 measurement windings A, B, and C

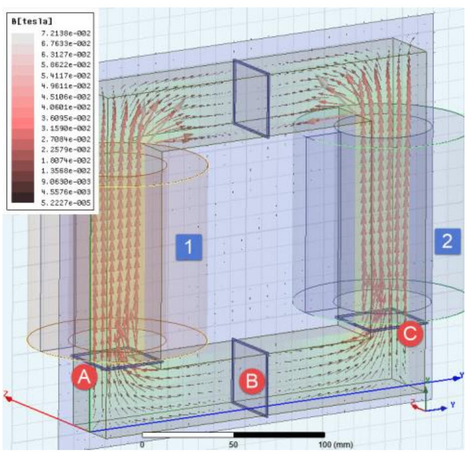


Fig. 2 The 3D FEA model for leakage inductance L_{12}
The primary and secondary coils are connected in opposite

IV. Experiments with the Model Transformer

For experiments, we use a core type transformer (fig.1 or 2) with similar primary and secondary windings of 380 turns each, equipped by three small measuring coils with 19 turns each. This transformer has unusually large leakage inductance, so the effect of the load variation on the core flux appears clearly. The goal of the experiments was evaluating the applicability of the equation (1), as well as comparing the accuracy the 2D and 3D FEA models for calculation of the mutual leakage inductance $M_{\sigma 123}$. The primary voltage varied in a wide range from the linear mode to highly saturated mode.

Neglecting the coil resistance, the mutual reactance $M\sigma 123$ can be directly measured by the short-circuit test of the transformer 1-2 as:

$$X_{123} = \frac{E_{3idle} - E_{3sc}}{I_2 sc}, \qquad (3)$$

where E_{3sc} are the EMF induced on open terminals of coil 3 in the no-load and short-circuit tests of the transformer 1-2 respectively, I_{2sc} – is the short-circuit current of the coil 2.

Both measured and calculated values of the mutual leakage reactance are given in table 1 (Ohms):

TABLE 1
Modeled and Measured Mutual Leakage Reactance

	3D FEA	2D FEA	Measured
Measuring winding.A	0.116	0.108	0.11
Measuring winding B	0.593	0.639	0.587
Measuring winding C	1.07	1.18	1.09

The measured (dashed) and calculated (solid) values of flux density in the core vs. load current are given on the fig. 3. The flux is given relative to the rated idle flux.

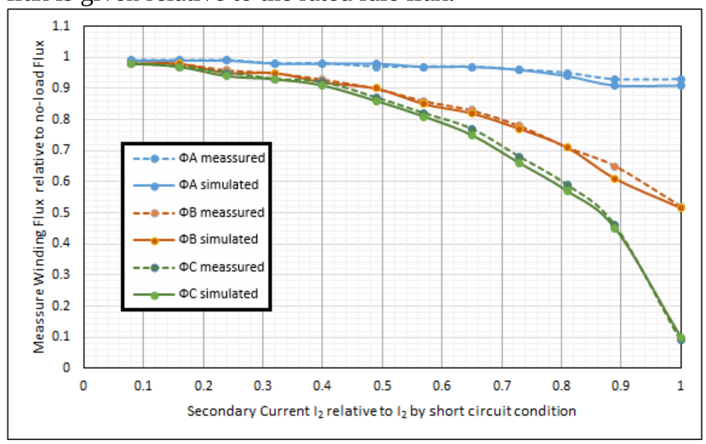


Fig.3. Comparing the measured (dashed) and calculated via 3D FEA (solid) flux in measuring coils A,B, and C vs. the load current..

The difference of calculated flux from measurement value is less than 1% for 3D FEA model and about 10% for 2D model. The agreement between simulated and measured data validates the assumptions and simplification described above. The results are valid over the full range of the primary voltage, because the magnetizing current remains small compared with the primary current. The curve Φ_C in fig. 3 shows that the cross section of the right leg of the magnetic core can be reduced, when the transformer is only energized on load condition. It is correct for wide range of spot welding transformers.

We can conclude that for design purposes simple 2D statics FEA simulation gives quite adequate results for identifying the core flux density in wide range of transformer load.

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